

# PATENT SPECIFICATION

(11) 1 297 779

## DRAWINGS ATTACHED

- (21) Application No. 60362/69 (22) Filed 10 Dec. 1969  
 (45) Complete Specification published 29 Nov. 1972  
 (51) International Classification H03H 7/00; H01P 5/14  
 (52) Index at acceptance  
       H3U 10B4 12D1 2A2A 3A  
       H1W 10A1 10AY 14X 14Y 19 5  
 (72) Inventor JOSEPH D. CAPPUCCI



## (54) NETWORKS USING A PLURALITY OF QUADRATURE COUPLER SECTIONS

(71) We, MERRIMAC RESEARCH AND DEVELOPMENT, INC., a corporation organized and existing under the laws of the State of New York, United States of America, of 41 Fairfield Place, West Caldwell, New Jersey 07007, United States of America, do hereby declare the invention for which we pray that a patent may be granted to us, and the method by which it is to be performed to be particularly described in and by the following statement:—

This invention relates to an electrical network employing a plurality of quadrature coupler sections.

In United States Patent Specifications Numbers 3,452,300 and 3,452,301 quadrature couplers are disclosed which are formed of one or more sections, each section in itself being a symmetrical directive coupler with imposed conditions of duality. In the first of the foregoing specifications, the couplers are of the multi-section type using lumped constant, frequency responsive components, i.e. capacitors and inductances. In the second specification, single and multi-section couplers are disclosed using an inductance formed by two wires which are highly magnetically coupled and held in registration with each other. The main part of the capacitance for this latter type of coupler is provided by the inter-winding capacitance between the two wires of the inductance. Further, these couplers use inductive coupled wires which are considerably less than a physical quarter-wave length long at the centre point of the operating band. Due to the use of the inter-winding capacitance, and the relatively small amount of wire needed to produce the needed inductance, couplers can be constructed which are extremely small in size, the size being determined primarily by the amount of wire needed to provide the inductance. In the couplers described in both of these specifications the coupler section or sections, operate as backward scattering type couplers.

According to the present invention there is

provided an electrical network including: first and second symmetrical, self-dual, four-port quadrature coupler sections which have different centre operating frequencies and each of which provides signals in quadrature at first and second of said ports in response to an input signal applied to either of the third or fourth of said ports; and means connecting the first and second ports of said first coupler section to the third and fourth ports respectively of said second coupler section.

Networks according to the invention may be designed as coupler networks or filters. Where a coupler network is required the above sections have at least one 180° phase shifting means inserted between them. Where filter networks are to be produced, a phase shift element is or is not used depending upon the network function desired. Networks can be designed with any number of cascaded quadrature coupler sections, the connecting means between successive sections being selected in accordance with the frequency response of the networks. Excellent couplers may be designed having well controlled coupling properties over a substantially wide band of operation. Further, pairs of transmission lines of arbitrary length, including zero length, can be used to connect the individual sections together without affecting the response function. The latter also holds true with respect to filter networks constructed in accordance with the invention. Further, the network synthesis made possible by the teachings of the present invention set out in greater detail below permits a wide variety of network functions to be synthesized with relative simplicity.

The present invention may be better understood by the description of preferred embodiments illustrated in the accompanying drawings in which:

FIGURE 1 shows a generalised four port, symmetric, directive coupler network in block form;

FIGURE 2 is a schematic diagram of one

form of a four port symmetric network according to Figure 1;

FIGURE 3 is a schematic diagram of a quadrature coupler section derived from Figure 2 but shown in a manner which better illustrates its application to networks of the present invention;

FIGURE 4 is a schematic diagram of coupler network having two quadrature coupler sections in accordance with the present invention;

FIGURE 5 is a schematic diagram of a network using three quadrature coupler sections in accordance with the present invention;

FIGURES 6A and 6B are graphical representations of the odd and even polynomial functions generated by coupler networks built in accordance with the invention; and

Fig. 7 is a schematic diagram of a filter network constructed in accordance with the invention.

In the following description the symbol  $W$  is used with the same meaning as the more usual symbol  $\omega$  to denote angular velocity (in radians/sec).

Figure 1 shows in general block notation a symmetric four port directive coupler section 10 having components (not shown) which are frequency responsive. The symmetric network has four ports 1, 2, 3 and 4 and is of the type such that when an input signal is applied to port 1, a coupled output signal is produced at port 2, a transmitted output signal is produced at port 3 in phase quadrature with the signal at port 2, and port 4 is isolated so that no output signal appears thereon. The network is symmetrical about both a transverse symmetry plane 11 and a longitudinal symmetry plane 12. Many such networks are well known in the art.

Since network 10 is symmetrical it can be analyzed, according to one theory, about the longitudinal plane of symmetry 12, in terms of even mode and odd mode bisections of the networks and their equivalent circuits. In the even mode bisection a voltage of  $+1/2V$  and  $+1/2V$  is considered to be applied to ports 1 and 2 and plane 12 is considered to be an open circuit reference to both input ports 1 and 2. In the odd mode bisection a voltage of  $+1/2V$  and  $-1/2V$  is considered to be applied to ports 1 and 2 and plane 12 is considered to be a short circuit reference to both input ports 1 and 2.

By imposing a condition of duality on the network 10 such that  $z_{in} = y_{in}$ , where  $z_{in}$

is the normalized input impedance for the even mode bisection of network 10 (where the normalized input impedance equals the input impedance of the network at any one frequency divided by the characteristic input impedance of the network) and  $y_{in}$  is the

normalized input admittance for the odd mode bisection of network 10 (where the normalized input admittance equals the input admittance at a specific frequency multiplied by the characteristic input impedance of the network). It can be shown by network analysis that the scattering coefficients for the symmetric network 10 are:

$$\begin{aligned} (1) \quad & S_{11} = 0 \\ (2) \quad & S_{12} = \Gamma_o \\ (3) \quad & S_{13} = [1 - \Gamma_o^2]^{1/2} \\ (4) \quad & S_{14} = 0 \end{aligned} \quad 75$$

Where  $V$  is the input voltage and  $\Gamma_o$  is the reflection coefficient in the even mode bisection, and  $[1 - \Gamma_o^2]$  is equal to  $T_o^2$ , where  $T_o$  is the transmission coefficient in the even mode. Equations (1) through (4) define a class of networks which are called "backward scattering directional couplers".

With an input  $V$  to port 1, equations (1) through (4) completely define the symmetric network 10 of Fig. 1 as a directional coupler having the following characteristics:

- (a) isolation (between ports 1 and 4). Since  $S_{14} = 0$  there is no signal transmission between ports 1 and 4.
- (b) input match (at port 1). Since  $S_{11} = 0$  there is no mismatch at the port 1 input.
- (c) coupled output (between ports 1 and 2) of  $V\Gamma_o$ . Since  $S_{12} = \Gamma_o$  defines the coupling between ports 1 and 2.
- (d) transmission output (between ports 1 and 3) of  $V[1 - \Gamma_o^2]^{1/2}$ . The term  $S_{13} = [1 - \Gamma_o^2]^{1/2}$  defines the transmission between ports 1 and 3.

The coupled output at port 2 can be shown to be in phase quadrature with the transmitted output at port 3. All of the foregoing is described in detail in the aforementioned specifications.

One type of symmetrical dual, directive network of the type discussed above with respect to Fig. 1 can be constructed in the manner shown in Fig. 2. The ports illustrated are each shown as a single terminal it being understood that the voltages thereat are all referred to a common ground connection. Here, the inductance 21 is of the two conductor type with one end of each wire being connected to a respective port of the network. Ports 1 and 2 are shown shunted by a capacitance 23 and ports 3 and 4 shunted by a capacitance 24. The conductors of the inductance 21 are highly magnetically coupled to each other. They can be wound as a bifilar winding on a toroid or coil form of high permeability magnetic material. They also can be twisted together or otherwise held in registration. In this case a high permeability core also can be used, if desired. As one way

of holding the two wires in registration, they can be printed on a printed circuit board or deposited on a substrate using suitable thin film or micro-circuit techniques.

5 The capacitance represented by the elements 23 and 24 can be of the lumped constant type. At high frequencies, where registered conductors are used to form the inductance, the interwinding capacitance is usually sufficient to form all, or substantially all, of the  
10 needed capacitance.

The symmetrical, directive network of Fig. 2 can be analyzed by the bisection technique and shown to have the following properties:  
15

$$(5a) \quad S_{12} = \frac{jx}{2+jx}$$

$$(5b) \quad S_{13} = \frac{2}{2+jx}$$

$$(5c) \quad S_{11} = S_{14} = 0$$

In the coupler described by equations (5a) and (5b) and shown in Fig. 2, the conditions of duality are imposed and it is assumed that the inductance 21 is of the bifilar type. In bifilar type inductances the odd mode inductance is significantly less than the even mode inductance and can, in most cases, be disregarded. The even mode inductance  $L_e$  is obtained from a shunt connection of the two conductors forming the total inductance, where the total parallel connected inductance is  $L_s$  so that  
20  
25  
30

$$L_e = 2L_s \quad (5.1)$$

In an even mode bisection, the capacitance value does not appear since the conductors are excited in parallel, so that no capacity can exist between them. In an odd mode bisection, the odd mode capacity  $C_o$  is twice the total capacity between the conductors  $C_s$ , since  $C_o$  is the capacity of one conductor to the short circuit reference 12. In most cases the odd mode inductance is small enough to be neglected.  
35  
40

Imposing the aforementioned condition of duality on the network such that the normalized input impedance for the even mode bisection is equal to the normalized input admittance for the odd mode bisection gives:  
45

$$Z_o W(C_o) = \frac{w(L_e)}{Z_o} \quad \text{or} \quad (5.2)$$

$$\frac{L_e}{C_o} = Z_o^2 \quad (5.3)$$

Thus in equations (5a) and (5b)

$$x = \frac{wL_e}{Z_o} \quad (5.4) \quad 50$$

where:

$L_e$  = the even mode inductance of one of the conductors from ports 1 to 2 or from ports 3 to 4.

$Z_o$  = the characteristic impedance of the system in which the network operates. 55

The derivation of these equations is explained in greater detail in United States Patent Specification No. 3,542,300.

Because the network of Fig. 2 is symmetric about both vertical and horizontal planes 11 and 12, the following relationships hold:  
60

$$(5d) \quad S_{12} = S_{21} = S_{34} = S_{43} = \frac{jx}{2+jx}$$

$$(5e) \quad S_{13} = S_{31} = S_{24} = S_{42} = \frac{2}{2+jx}$$

$$(5f) \quad S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad 65$$

$$(5g) \quad S_{14} = S_{41} = S_{23} = S_{32} = 0$$

Equations 5d through 5g are defined by a scatter matrix analysis which is reproduced below:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{13} \\ S_{13} & 0 & 0 & S_{12} \\ 0 & S_{13} & S_{12} & 0 \end{bmatrix} \quad 70$$

To form one type of symmetrical quadrature coupler network section which can be used in the networks of the present invention, the dual, symmetric network of Fig. 2 is first rotated schematically counter-clockwise by 90°. Starting from the upper left-hand corner the port numbers, in a clockwise direction, now becomes 3, 4, 2, 1. The ports 3 and 4 are then interchanged by lifting off the respective connected end of each of the windings of the inductor 21 and connecting them to the other port (i.e. 3 to 4 and 4 to 3). Flipping the resultant network in its entirety by 180° about its horizontal center line gives the network 20 shown in Fig. 3. The windings 21a and 21b in Fig. 3 correspond to the windings of the bifilar inductance 21 of Fig. 2.  
75  
80  
85

The transformation of the coupler of Fig. 2 to that of Fig. 3 makes (schematically) the quadrature coupler section 20 of Fig. 3 a symmetric "forward scatter" coupler as distinguished from the symmetric "backward scatter" coupler of Fig. 2. However, this dis-  
90

inction is made to help aid in visualizing the coupler section in physical arrangement and signal flow terms in the later description rather than in implying any difference in electrical performance. The symmetry referred to with respect to the coupler 20 of Fig. 3 is the port symmetry shown in equations (5d) through (5g). From equations (5d) through (5g) it should be obvious that any port of the coupler 20 can be used as an input port with the output ports being on the opposite side of the coupler and the port adjacent to the input port being isolated. The application of an input signal to either port on one side of the coupler will produce a pair of quadrature signals at the ports on the other side. The network 20 of Fig. 3 still has the same scattering parameters given in equations (5a) through (5c). Also the same relationships hold for L and C as given above with respect to the network of Fig. 2.

Networks in accordance with the present invention, are constructable from a plurality of the "forward scatter" coupler sections shown in Fig. 3 which are connected together to produce circuit networks such as quadrature couplers and filters. Several quadrature coupler applications are discussed first.

Fig. 4 shows two of the coupler sections 20A and 20B of the type shown in Fig. 3 connected together by a phase inverting transformer 30 to form a quadrature coupler network 40. Each of the coupler sections 20A and 20B is of the same construction as the section 20 of Fig. 3 and the same reference numerals are used for the components with the respective A and B suffixes, the bifilar inductances being designated simply by reference numeral 21 with the appropriate suffix. Ports 1 and 4 of section 20A and ports 2 and 3 of section 20B form the respective input and output ports for the coupler 40. Depending upon whether port 1 or 4 is used as the input port, the other would be isolated from the input and usually, although not necessarily, terminated in the characteristic impedance of the system. The phase inverting transformer 30 can be of any suitable type such as a balun anti-balun pair. A separate transformer 30—1 is shown between port 2 of section 20A and port 1 of section 20B and transformer 30—2 between port 3 of section 20A and port 4 of section 20B. The transformer 30—1 produces the 180° phase reversal of the signal transmitted between ports 2 and 1 of the coupler sections 20A and 20B respectively, the transformer 30—2 producing no phase change other than residual phase shift. The transformer sections 30—1 and 30—2 have the same residual phase shift so that a 180° phase differential is maintained between them. Any suitable phase reversing, or shifting, component or network can be

used instead of the transformer 30, as long as the 180° phase differential is produced. For example, an LC network or one of the type described by B. N. Schiffman in IRE Transactions, Professional Group Microwave Theory and Techniques, April 1958, pages 232—237 "A New Class of Broad Band 90° Phase Shifters"; and S. B. Bedrosian in IRE Transactions Professional Group Circuit Theory June 1960, pages 128—136 "Normalized Design of 90° Phase Difference Networks".

From equations (5d) through (5g) coupler section 20A is described by the following scattering parameters:

$$(6) \quad S_{12A} = \frac{jwa}{2 + jwa}$$

$$(7) \quad S_{13A} = \frac{2}{2 + jwa}$$

$$S_{11A} = S_{14A} = 0$$

Coupler section 20B is described by:

$$(8) \quad S_{12B} = \frac{jwb}{2 + jwb}$$

$$(9) \quad S_{13B} = \frac{2}{2 + jwb}$$

$$S_{11B} = S_{14B} = 0$$

For coupler section 20A in equations (6) and (7),

$$a = \frac{L_A}{Z_0},$$

where  $L_A$  is the even mode inductance of one of the inductance windings of section 20A from ports 1 to 3 or 4 to 2 and  $Z_0$  is the characteristic impedance of the system in which coupler 40 operates.

Similarly in equations (8) and (9) for section 20B

$$b = \frac{L_B}{Z_0},$$

where  $L_B$  is the even mode inductance of one of the inductance windings of section 20B from ports 1 to 3 or 4 to 2 and  $Z_0$  is the characteristic impedance of the system in which coupler 40 operates.

For the total coupler network 40 shown in Fig. 3 the scattering parameters are :

$$(10) \quad S_{12} = S_{13A} S_{13B} - S_{12A} S_{12B}$$

$$(11) \quad S_{13} = S_{13A} S_{12B} - S_{12A} S_{13B}$$

5 and

$$S_{11} = S_{14} = 0$$

The minus sign between the two terms on the right-hand side of each of the equations (10) and (11) occurs due to the 180° phase reversal provided by the transformer 30. The terms  $S_{12}$  and  $S_{13}$  apply between the input port 1 and the output ports 2 and 3 of the complete two section network, respectively.

If the phase shift from port 2A to port 1B is defined as  $\theta + 180^\circ$  and the phase shift from port 3A to port 4B is defined as  $\theta$ ,

where  $\theta$  is the residual phase of the transformers plus the interconnecting line lengths, then equation (10) can be written as:

$$20 \quad S_{12} = S_{12A} e^{j(\theta+180^\circ)} S_{12B} + S_{13A} e^{j\theta} S_{13B} \quad (10a)$$

and equation (11) as:

$$S_{13} = S_{13A} e^{j\theta} S_{12B} + S_{13A} e^{j(\theta+180^\circ)} S_{13B} \quad (11a)$$

Equations (10a) and (11a) can be rewritten as:

$$25 \quad S_{12} = (S_{13A} S_{13B} - S_{12A} S_{12B}) e^{j\theta} \quad (10b)$$

$$S_{13} = (S_{13A} S_{12B} - S_{12A} S_{13B}) e^{j\theta} \quad (11b)$$

From equations (10b) and (11b) it is seen that each output is equally shifted in phase and the line length  $\theta$  is just a reference shift of both outputs having no effect on the phase differential between the outputs nor on the amplitude response of either output.

This condition holds for any number of coupler sections interconnected and, as such, will not be included in any further discussions.

Equations (10a), (10b), (11a) and (11b) show that the entire coupler is independent of the interconnecting line lengths between the respective coupler sections so long as the line lengths between adjacent sections are paired, i.e. have the same phase shift. This means that the pairs of line lengths can be of any arbitrary value, including zero. Also, the phase shift of a pair of line lengths can be selected to produce a desired overall phase shift for a given network. Successive pairs of line lengths between adjacent coupler sections can have different phase shifts. In essence, this means

that in the networks of the present invention that line lengths, and their resultant phase shift, are chosen as a matter of convenience rather than being imposed upon the network as a constraint. This gives the circuit designer greater flexibility.

Substituting equations (6), (7), (8) and (9) into equations (10) and (11) and neglecting the residual phase shift between sections gives :

$$(12) \quad S_{12} = \frac{4 + w^2 ab}{(4 - w^2 ab) + jw(2a + 2b)}$$

and

$$(13) \quad S_{13} = \frac{jw(b-a)}{(4 - w^2 ab) + jw(2a + 2b)} \quad 60$$

Since the Denominators at the equations for  $S_{12}$  and  $S_{13}$  are identical, and the numerators are in phase quadrature, then the phases at  $S_{12}$  and  $S_{13}$  are always in phase quadrature.

The loss function  $L$  between port 1 of coupler 40 and any other port is given by

$$L = \frac{1}{|S|^2}$$

Between ports 1 and 2, from equation (12) the loss function  $L_{12}$  becomes:

$$(14) \quad L_{12} = \frac{16 + w^4 a^2 b^2 - 8w^2 ab + w^2(4a^2 + 4b^2 + 8ab)}{16 + 8w^4 ab + w^4 a^2 b^2}$$

$$L_{12} = 1 + \frac{w^2(4a^2 + 4b^2 - 8ab)}{16 + 8w^4 ab + w^4 a^2 b^2}$$

$$L_{12} = 1 + \left\{ \frac{2w(b-a)}{4 + w^2 ab} \right\}^2$$

Similarly, using (13), the loss function  $L_{13}$  between ports 1 and 3 of coupler 40 is expressed as

$$(15) \quad L_{13} = 1 + \left\{ \frac{4 + w^2 ab}{2w(b-a)} \right\}^2$$

It should be noted that the numerator of (14) equals the denominator of (15) and the denominator of (15) equals the numerator of (14).

Rewriting  $L_{13}$ , as given by (15), in a slightly different form gives:

$$(16) \quad L_{13} = 1 + \left\{ \frac{2}{w(b-a)} + \frac{wab}{2(b-a)} \right\}^2$$

To make the function  $L_{13}$  have geometric symmetry in  $w$  (i.e.  $f(w)=f(1/w)$ ) then  $ab=4$ . Equation (16) now becomes

$$(17) \quad L_{13} = 1 + \frac{(L_c)^2}{(b-a)^2} \left( \frac{w-1}{w} \right)^2$$

5 The function  $L_{13}$  of equation (17) has its minimum value at  $w=1$ , and at  $w=1$ ,  $L_{13}$  becomes

$$(18) \quad L_{13} = 1 + \frac{(L_c)^2}{(b-a)^2}$$

10 The coupling  $C$  between ports 1 and 3, called  $C_{13}$ , of coupler 40 is expressed in db as:

$$(19) \quad C_{13} \text{ (db)} = 10 \log_{10} [L_{13}].$$

As an example, for a 3db coupler at  $w=1$

$$\frac{1}{b-a} = 1 \quad \text{and} \quad ab=4$$

15 Since, for geometric symmetry in  $w$ ,  $ab=4$ , then

$$b = 2[1 + \Gamma^2] = 4.82843 \quad (19a)$$

and

$$a = \frac{2}{(1 + \Gamma^2)} = .82843 \quad (19b)$$

20 Since  $a$  and  $b$  are the normalized inductive reactance values of the section 20A and 20B at  $w=1$ , the geometric center of a respective section, the inductance values  $L_A$  and  $L_B$  of the inductance 21 for the two sections can be readily calculated. Also, due to the original constraint of duality imposed upon the sections, the capacitance value for the capacitors 23 and 24 are given as, in bisection

$$\sqrt{\frac{L}{C}} = Z_0$$

30 It should be recalled that the values of  $L$  and  $C$  are the bisection values given in equations (5.2) and (5.3).

The loss function  $L_{13}$ , as a function of  $w$  (for 3db coupling at  $w=1$ ) becomes:

$$(20) \quad L_{13} = 1 + \left( \frac{w^2 - 1}{2w} \right)^2 \quad 35$$

Any coupling value can be synthesized by inserting the proper value for  $b-a$  in equation (18) to get the desired coupling and solving for  $a$  and  $b$  from,

$$(21) \quad b - a = k \quad 40$$

$$(22) \quad ab = 4$$

Where  $|k|$  is greater than zero.

From (19a) and (19b) it is seen that the values of  $a$  and  $b$  are different for a 3 db coupler. This means that the section 20A and 20B will have different cross-over frequencies, i.e. the cross-over frequency being the point where  $|S_{12}| = |S_{13}|$ . 45

Quadrature couplers having more than two sections can also be synthesized and constructed using the techniques described above. Fig. 5 shows a quadrature coupler network 50 using three sections of the type 20 shown in Fig. 3, designated 20A, 20B and 20C. The two transformers 30-1 and 30-2 which provide a 180° relative phase shift between the signals provided on the ports 2 and 3 of the section 20A are connected respectively between port 2 of section 20A and port 1 of section 20B and port 3 of section 20A and port 4 of section 20B. There is a connection between port 2 of section 20B and port 1 of section 20C and port 3 of section 20B and port 4 of section 20C without a 180° phase inversion. The input ports for the complete coupler are ports 1 and 4 of section 20A while the output ports are ports 2 and 3 of section 20C. 50 55 60 65

The scattering parameters for section 20C are given as: 70

$$(23) \quad S_{12} = \frac{jwc}{2 + jwc}$$

$$(24) \quad S_{13} = \frac{2}{2 + jwc}$$

$$S_{11} = S_{11c} = 0$$

For coupler section 20C in equations (23) and (24)

$$c = \frac{L_c}{Z_0}$$

where  $L_c$  is the series inductance of one of the

the inductance windings of section 20C from ports 1 to 3 or 4 to 2 and  $Z_0$  is again the characteristic impedance of the system. The scattering parameters and the values for  $a$  and  $b$  of sections 20A and 20B remain as previously described in equations (6) through (9).

The scattering parameters for the total coupler network 50 of Fig. 5 are:

$$(25) \quad S_{12} = S_{12c} [S_{13A} S_{13B} - S_{12A} S_{12B}] + S_{13c} [-S_{12A} S_{13B} + S_{12B} S_{13A}]$$

$$(26) \quad S_{13} = S_{12c} [-S_{12A} S_{13B} + S_{12B} S_{13A}] + S_{13c} [S_{13A} S_{13B} - S_{12A} S_{12B}]$$

$$15 \quad \text{and } S_{11} = S_{14} = 0.$$

The minus signs in the terms of equations (25) and (26) are again due to the  $180^\circ$  phase shift produced by the network 30.

For the entire coupler 50, substituting  $S_{12c}$ ,

$$20 \quad S_{12c}, S_{12c}, S_{13c}, S_{13c} \text{ and } S_{13c} \text{ e.g. (25) and (26) gives:}$$

$$(27) \quad S_{12} = \frac{4w(b+c-a) \cdot w^2 abc}{8-2w^2(ab+bc+ac) + j[4w(a+b+c) - w^2 abc]}$$

$$(29) \quad |D|^2 = [8 + 2w^2(ab+ac-bc)]^2 + [w^2 abc + 4w(b+c-a)]^2$$

For the three section coupler the loss function  $L = \frac{1}{|S|^2}$  for the ports 1 to 2 and 1 to 3 becomes from equations (27) and (28):

$$35 \quad L_{12} = 1 + \frac{[8-2w^2(ab+bc+ac)]^2}{4w^2(b+c-a) \cdot w^2 abc}$$

$$(31) \quad L_{13} = 1 + \frac{[4w(b+c-a) \cdot w^2 abc]^2}{8-2w^2(ab+bc+ac)}$$

The second term of the right-hand side of both equations (30) and (31) for  $L_{12}$  and  $L_{13}$  are polynomials which can be designated as being of the general type  $P_{in}^2(w)$ . In the case

and

$$(28) \quad S_{13} = \frac{8-2w^2(ab+bc+ac)}{8-2w^2(ab+bc+ac) + j[4w(a+b+c) - w^2 abc]}$$

$S_{12}$  and  $S_{13}$  are again in phase quadrature since their denominators are identical and their numerators are in phase quadrature.

Squaring the denominator of each of equations (27) and (28) produces a quantity  $|D|^2$  which can be factored into:

of the three section network the function  $P_{in}^2(w)$  can be made antisymmetric about  $w=1$  so that:

$$(32) \quad P_{in}^2(w) = \frac{1}{P_{in}^2(1/w)}$$

In equation (32), it should be noted from (30) and (31) that:

$$(33) \quad L_{in}(w) = 1 + P_{in}^2(w)$$

Solving (30) and (31) to get the anti-symmetric condition of (32) gives:

$$(34) \quad abc = 8 \text{ and}$$

$$(35) \quad 4(b+c-a) = 2(ab+ac-bc)$$

5

10

25

30

50

Solving (34) and (35) for  $a$  gives:

$$(36) \quad a^2(2+b+c) - 2a(b+c) - 8 = 0$$

which yields

$$(37) \quad a = \frac{(b+c) \pm (4+b+c)}{2+(b+c)}$$

5 Since  $a$ ,  $b$  and  $c$  must be positive quantities, one must choose the positive root, then

$$a=2$$

Substituting the value of  $a=2$  and  $abc=8$  into equations (30) and (31) gives:

$$10 \quad L_{12} = 1 + \frac{(38) \quad [4 \cdot 2w^2(b+c-2)]^2}{[2w(b+c-2) + w^3bc]^2} \quad \text{and} \quad L_{13} = 1 + \frac{(39) \quad [2w(b+c-2) + w^3bc]^2}{[4 \cdot 2w^2(b+c-2)]^2}$$

where  $a=2$  and  $bc=4$ .

The quantities  $b$  and  $c$  can be solved for by conventional technique. For example, the polynomial portion of (38) can be differentiated and set equal to zero. Solving for  $b+c$  as a function of  $w$  we obtain

$$b+c = \left(\frac{w+1}{w}\right)^2 \cdot \left[\left(w^2 + \frac{1}{w^2}\right)^2 - \frac{1}{w^2}\right]^{\frac{1}{2}}$$

and the maximum and minimum values of the loss functions can be obtained by substituting the values obtained for  $b$  and  $c$  at the value of  $w$  chosen. (Since the function has geometric anti-symmetry about  $w=1$  the loss function at 1

— is also known). Through the use of frequency scaling factors, the inductance for  $a$ ,  $b$  and  $c$  can be scaled to the correct center frequency, and the capacitance can be found from (5.2) and (5.3).

It should again be noted that  $a$ ,  $b$  and  $c$  are different quantities. This means that each coupler section 20A, 20B and 20C has a different center frequency.

As should be apparent, the synthesis of two systems of couplers has been described using quadrature couplers of the forward scatter type. The first being a system of even numbers of coupler sections, the second being a system of odd numbers of coupler sections. In both systems the loss functions  $L_{12}$  and  $L_{13}$  are always expressed by:

$$(40) \quad L_{12} = 1 + \frac{|S_{13}|^2}{|S_{12}|^2} \quad 40$$

and

$$(41) \quad L_{13} = 1 + \frac{|S_{12}|^2}{|S_{13}|^2}$$

The polynomials,  $(P_{2n}(w))$  for the couplers formed by an even number of sections will always have geometric symmetry about  $w=1$ . And  $S_{12}$  and  $S_{13}$  are always in phase quadrature. The odd polynomials  $(P_{(2n-1)}(w))$  for the couplers formed by an odd number of sections will always have geometric anti-symmetry about  $w=1$ . The  $w$  quantities are normalized frequencies.

For couplers with an even number of sections, the polynomial will be of the following form:

$$(42) \quad P_{(2n)}(w) = \frac{M_1 \cdot M_3 w^2 + \dots + M_{(2i-1)} w^{2i} + \dots + M_{(2n-1)} w^{2n}}{M_2 w + \dots + M_{2i} w^{(2i-1)} + \dots + M_{2n} w^{(2n-1)}} \quad 55$$

where  $i$  goes from 1 to  $n$  and  $M_1 = M_{(2n-1)}$ ,  $M_3 = M_{(2n-3)}$ ,  $M_{(2i-1)} = M_{(2(n-i)+1)}$ , and  $M_2 = M_{2n}$ ,  $M_4 = M_{(2n-2)}$ ,  $M_{2i} = M_{2(n-i+2)}$ . Where  $n$  is the order of the polynomial, (i.e.  $n=1, 2, 3, \dots$ ) and  $i$  is index of the term within the polynomial, and the number of elements required in  $2n$ .

For couplers with an odd number of sections, the polynomial will be of the following form:

$$(43) \quad P_{(2n-1)}(w) = \frac{M_1 w + M_3 w^3 + \dots + M_n w^{(2n-1)}}{M_n M_{(n-1)} w^2 + \dots + M_1 w^{2(n-1)}} \quad 65$$

where  $n$  is the order of the polynomial, (i.e.  $n=1, 2, 3, \dots$ ) and the number of elements required is  $= (2n-1)$ .

Figs. 6A and 6B respectively show, diagrammatically, the polynomials of the form  $P_{2n}$  and  $P_{(2n-1)}$ . As should be apparent from the form of the curves of these two figures and equations (42) and (43), which describe the even and odd number section couplers, the polynomials are of the rational Tschebyscheff type. It also should be apparent from (38) and (39) and the more generalized counterpart corresponding equations (40) and (41)



that the polynomial portions of the equations have poles and zeros on the imaginary axis. This again verifies the conclusion that the networks constructed with at least one 180° phase shifting network between two coupler sections is a coupler.

It should be understood that while only one 180° phase shifting network 30 is shown in the networks of both Figs. 4 and 5, that the more general statement holds that in any network of N coupler sections (e.g. the sections 20) up to N-1 180° phase shifting networks (e.g. 30) can be used. This also includes even numbers of 180° phase shifting networks. The order of interconnection of the sections 20 determines the number of 180° phase shifters, needed. At least one 180° phase shifter is needed to form a quadrature coupler network.

The Polynomials described above are all 3 db couplers in the form analyzed. If the polynomials were rewritten in the form

$$(44) \quad P_1(w) = kP(w)$$

and the elements recalculated, it can produce a coupler of any coupling value.

The networks heretofore described are of the coupler type with at least 180° phase shifting network. Fig. 7 shows another type of network formed by two coupler sections 20A and 20B of the type shown in Fig. 3 in which the sections are connected together without such a 180° phase shifting section. Here again, the coupler sections have a different center frequency. It can be shown by an analysis similar to that previously presented that the network of Fig. 7 will produce polynomial terms in the loss function where all  $w$ ,  $w^2$ ,  $w^4$ ,  $w^6$  . . . etc. terms are negative. It also can be shown that the poles and zeros of these polynomial terms lie on the real axis. This means that such networks are filters. These filters are of the directional type, i.e. the filter has two outputs. They also have a pass band and a stop band with one or more of such pass bands and stop bands being possible.

While it has been found preferable to construct filter networks according to the present invention without the use of 180° phase shifting networks, filters can be constructed using at least one such phase shifting network if there is a proper choice of the  $a$ ,  $b$ ,  $c$  . . . etc. parameters.

The networks described previously use coupler sections 20 which are formed by elements at least one of which is lumped, that is, the inductance. However, the coupler sections do not have to be of this form. By proper insertion of the number and location of 180° sections, some are all of the real axis poles (or zeros) can be moved to the imaginary axis, leaving only those zeroes (or poles) on the real axis that are desired, forming band-

pass, high-pass, or low-pass functions with either rippled or monotonic passbands, or stop bands, as desired.

The coupler sections can be of any forward scatter directional type, and the equations of (10) and (11) still hold, so that the loss function is still definable as 1 plus the ratio of  $S_{12}$  to  $S_{13}$  squared. In many cases this will not produce the geometric symmetry of the case cited, but will always produce a useful coupling structure.

Examples of other types of couplers that can be used are: short-slot coupler sections in waveguide form connected by a pair of waveguides, one of which is twisted 180°, can form a coupler network: quarter-wavelength backward couplers can be used with 180° sections to form a useful coupler section: any aperture coupled waveguide coupler section can be interconnected with a 180° twist to form a useful coupler network. The same holds true for the coupler sections forming the filter networks.

#### WHAT WE CLAIM IS:—

1. An electrical network including: first and second symmetrical, self-dual, four-port quadrature coupler sections which have different center operating frequencies and each of which provides signals in quadrature at first and second of said ports in response to an input signal applied to either of the third and fourth of said ports;

and means connecting the first and second ports of said first coupler section to the third and fourth ports respectively of said coupler section.

2. A network as defined in Claim 1 wherein said connecting means includes means for producing a relative 180° phase shift between signals appearing on said third and fourth ports of said second coupler section thereby providing a network in which the polynomial terms of the loss functions from the third port of the first coupler section to the first and second ports of the second coupler section have their poles and zeros on the imaginary axis rendering the network a quadrature coupler network.

3. A network as defined in Claim 2 wherein said connecting means comprises a balun anti-balun pair.

4. A network as defined in Claim 2 in which:

The characteristic impedance of said first and second coupler sections is  $Z_0$ ; where

$$Z_0 = \frac{L_1}{C_1} = \frac{L_2}{C_2} \quad 120$$

$L_1$  and  $L_2$  being one-half the even mode inductances of said first and second coupler sections respectively and  $C_1$  and  $C_2$  being

one-half the odd mode capacitances of said first and second coupler sections respectively.

5. A network as defined in Claim 4 in which:

$$\frac{\omega_o^2 L_1 L_2}{Z_o^2}$$

5

is equal to four so that the network is symmetrical with respect to  $\omega_o$ .

6. A network as defined in Claim 4 wherein said first and second couplers each comprises a pair of registered conductors tightly magnetically coupled to minimize leakage reactance;

and capacitive coupling therebetween to provide said characteristic impedance of  $Z_o$ .

7. A network as defined in Claim 3 also including:

a third symmetrical, four-post quadrature coupler section which is self dual and has a different center operating frequency than said first and second coupler sections for providing signals in quadrature at first and second of said ports in response to an input signal applied to either of the third and fourth of said ports;

and means for connecting said first and second ports of said second coupler section to said third and fourth ports of said third coupler section respectively.

8. A network as defined in Claim 7 in which the characteristic impedance of said first, second and third coupler sections is  $Z_o$ ; where

$$Z_o = \frac{L_1}{C_1} = \frac{L_2}{C_2} = \frac{L_3}{C_3},$$

$L_1$ ,  $L_2$ , and  $L_3$  being one-half the even mode inductances of said first, second and third

35

coupler sections respectively and  $C_1$ ,  $C_2$ , and  $C_3$  being one-half the odd mode capacitances of said first, second and third coupler sections respectively; and

$$\frac{\omega_o^3 L_1 L_2 L_3}{Z_o^3}$$

40

is equal to eight so that the network is symmetrical with respect to  $\omega_o$ .

9. A network as defined in Claim 7 or 8 in which said first and second ports of each of said first, second, and third coupler sections are on the same side of said first, second and third couplers respectively.

10. A network as defined in Claim 7 also including:

a plurality of additional symmetrical four-port quadrature coupler sections each of which is self dual and has a different center operating frequency from the or each other additional section and from said first, second and third coupler sections for providing a signal in quadrature signal at first and a second of said ports in response to an input signal applied to either of the third or fourth of said ports;

and means for connecting said plurality of additional coupler sections in cascade with one another and with said third coupler section.

11. A network as defined in Claim 10 in which said connecting means includes 180° phase shifting networks.

12. A network substantially as hereinbefore described with reference to Figure 4; Figure 5; or Figure 7 of the accompanying drawings.

TREGEAR, THIEMANN & BLEACH,  
Chartered Patent Agents,  
Melbourne House,  
Aldwych, London, W.C.2.,  
Agents for the Applicants.

FIG. 1

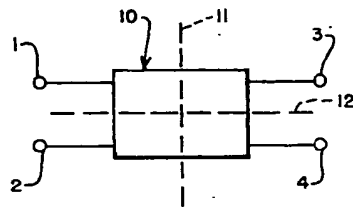


FIG. 2

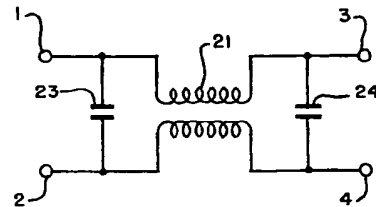


FIG. 4

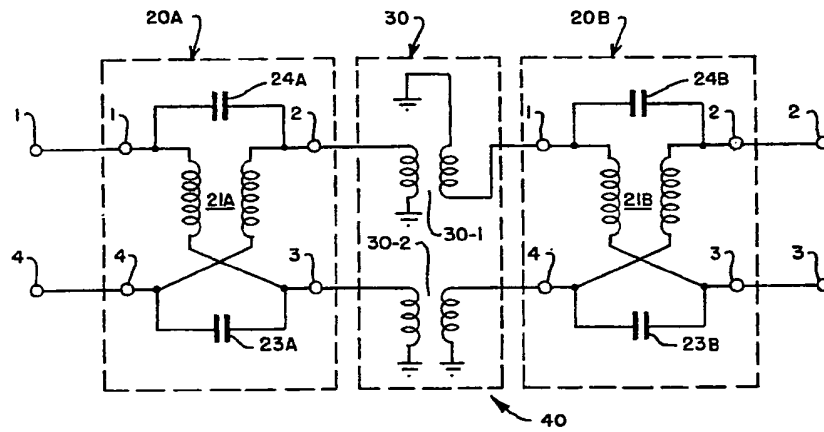


FIG. 3

